The Heathland School Mathematics Department
A Level Course Preparation Questions 2023

PRACTICE QUESTIONS BOOKLET
(complete all questions on paper)

In order to succeed on this course it is imperative that you start in September being proficient at a number of key skills from GCSE Mathematics. In order to ensure that this is the case, this question booklet consists of some high level GCSE practice questions, which you will need to complete fully. Numerical answers are given at the end so that you are able to tell if you have completed the questions correctly.

The expectation is that you are fully competent in applying these skills and will have the necessary perseverance to complete all questions and will be able to show relevant methods fully.

Teachers will be asking to see evidence of all work completed week beginning **Monday 11th September** and will be checking for full working out which shows complete understanding for all questions.

During your **first double** A Level lesson week beginning **Monday 11th September**, you will take a ***60 minute test*** to assess the topics covered in this booklet. Calculators may be used both in completing this booklet and the test but **method must be shown**.

All students who do not complete the booklet fully or who perform poorly in the test will then need to consider carefully whether they have the aptitude required to succeed on the demanding Mathematics A Level course.

There are notes and examples to help you from page 15 onwards

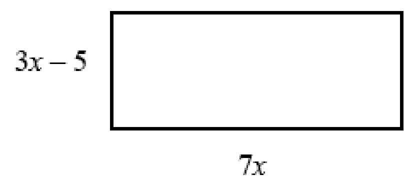
Practice questions - Expanding brackets and simplifying expressions

Q1) Expand and simplify where possible.

a $-2(5pq + 4q^2)$	b $8(5p - 2) - 3(4p + 9)$
c $-2h(6h^2 + 11h - 5)$	d $3b(4b - 3) - b(6b - 9)$
e $(2x + 3)(x - 1)$	f $(3x - 2)(2x + 1)$
g $(5x - 3)(2x - 5)$	h $(3x - 2)(7 + 4x)$
i $(3x + 4y)(5y + 6x)$	j $(x + 5)^2$
k $(2x - 7)^2$	l $(4x - 3y)^2$

Q2) Extend

a) The diagram shows a rectangle.
Write down an expression, in terms of x, for the area of the rectangle.
Show that the area of the rectangle can be written as $21x^2 - 35x$.



b) A cuboid has dimensions (x + 2)cm, (2x – 1)cm and (2x + 3)cm.
Show that the volume of the cuboid is $(4x^3 + 12x^2 + 5x - 6)cm^3$

Practice questions - Expanding brackets and simplifying expressions

Q1) Simplify			
a	$\sqrt{45}$	**b**	$\sqrt{48}$
c	$\sqrt{300}$	**d**	$\sqrt{72}$
e	$\sqrt{72}+\sqrt{162}$	**f**	$\sqrt{50}-\sqrt{8}$
g	$2\sqrt{28}+\sqrt{28}$	**h**	$2\sqrt{12}-\sqrt{12}+\sqrt{27}$
Q2) Expand and Simplify			
a	$(\sqrt{2}+\sqrt{3})(\sqrt{2}-\sqrt{3})$	**b**	$(4-\sqrt{5})(\sqrt{45}+2)$
Q3) Rationalise and simplify, if possible.			
a	$\frac{1}{\sqrt{5}}$	**b**	$\frac{2}{\sqrt{7}}$
c	$\frac{2}{\sqrt{2}}$	**d**	$\frac{\sqrt{8}}{\sqrt{24}}$
e	$\frac{2}{4+\sqrt{3}}$	**f**	$\frac{6}{5-\sqrt{2}}$
Q4) Extend			
Solve the equation $8+x\sqrt{12}=\frac{8x}{\sqrt{3}}$. Give your answer in the form $a\sqrt{b}$ where a and b are integers.			

Practice questions - Indices

Q1) Evaluate

a 14^0	b $125^{\frac{1}{3}}$
c $8^{\frac{5}{3}}$	d 2^{-5}
e $27^{-\frac{2}{3}}$	f $\left(\frac{9}{16}\right)^{-\frac{1}{2}}$
g $16^{\frac{1}{4}} \times 2^{-3}$	h $\left(\frac{27}{64}\right)^{-\frac{2}{3}}$

Q2) Simplify

a $\dfrac{10x^5}{2x^2 \times x}$	b $\dfrac{c^{\frac{1}{2}}}{c^2 \times c^{\frac{3}{2}}}$
c $\dfrac{y^2}{y^{\frac{1}{2}} \times y}$	d $\dfrac{\left(2x^2\right)^3}{4x^0}$

Q3) Write the following as a single power of *x*.

a $\dfrac{1}{x^7}$	b $\dfrac{1}{\sqrt[3]{x}}$
c $\sqrt[5]{x^2}$	d $\dfrac{1}{\sqrt[3]{x^2}}$

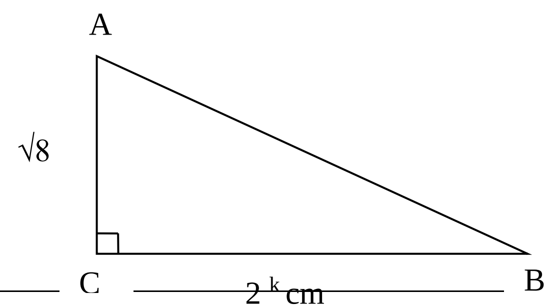
Q4) Write the following in the form ax^n.

a $\dfrac{2}{x^3}$	b $\dfrac{4}{\sqrt[3]{x}}$

Q5) Problem Solving

a) Triangle ABC has an area of 32 cm^2.

Calculate the value of k.



b) Given that $243\sqrt{3} = 3^a$, find the value of a.

Practice questions - Factorising

Q1) Factorise			
a	$25x^2y^2 – 10x^3y^2 + 15x^2y^3$	b	$18a^2 – 200b^2c^2$
c	$x^2 – 3x – 40$	d	$x^2 + 3x – 28$
e	$2x^2 + 7x + 3$	f	$12x^2 – 38x + 20$
Q2) Simplify the algebraic fractions			
a	$\frac{2x^2+4x}{x^2-x}$	**b**	$\frac{x^2-x-12}{x^2-4x}$
c	$\frac{x^2-5x}{x^2-25}$	**d**	$\frac{2x^2-7x-15}{3x^2-17x+10}$
e	$\frac{4-25x^2}{10x^2-11x-6}$	**f**	$\frac{(x+2)^2+3(x+2)^2}{x^2-4}$
Q3) Extend			
a) Express $\frac{1}{x-2}-\frac{2}{x+4}$ as a single algebraic fraction. **b)** Hence, or otherwise, solve $\frac{1}{x-2}-\frac{2}{x+4}=\frac{1}{3}$			

Practice questions – Quadratics

Q1) Solve by factorising	
a $x^2 + 7x + 10 = 0$	b $x^2 + 3x - 10 = 0$
c $2x^2 - 7x - 4 = 0$	d $3x^2 - 13x - 10 = 0$
Q2) Solve by completing the square	
a $x^2 - 10x + 4 = 0$	b $x^2 - 2x - 6 = 0$
c $2x^2 + 8x - 5 = 0$	d $5x^2 + 3x - 4 = 0$
Q3) Solve by using the quadratic formula, giving your solutions in surd form	
a $3x^2 + 6x + 2 = 0$	b $2x^2 - 4x - 7 = 0$
Q4) Problem Solving	

$x^2 - 14x + 1 = (x + p)^2 + q$, where p and q are constants.

a Find the values of p and q.

b Using your answer to part **a**, or otherwise, show that the solutions to the equation $x^2 - 14x + 1 = 0$ can be written in the form $r \pm s\sqrt{3}$, where r and s are constants to be found.

Q5) Extend

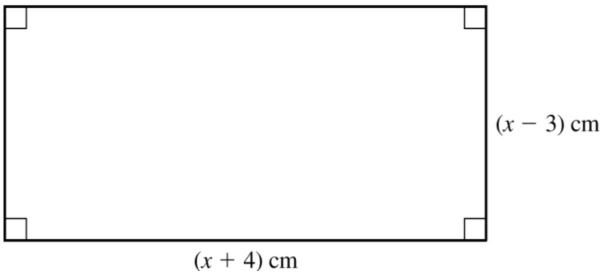


Diagram **NOT** accurately drawn.

The length of a rectangle is $(x + 4)$ cm. The width is $(x - 3)$ cm. The area of the rectangle is 78 cm^2.

(a) Use this information to write down an equation in terms of x.

(b) **(i)** Show that your equation in part **(a)** can be written as

$$x^2 + x - 90 = 0$$

(ii) Find the values of x which are the solutions of the equation

$$x^2 + x - 90 = 0$$

x = ... or x = ...

(iii) Write down the length and the width of the rectangle.

length = ... cm   width = ... cm

Practice questions – Sketching quadratics

1 Sketch the graph of $y = -x^2$.

2 Sketch each graph, labelling where the curve crosses the axes.

a $y = (x + 2)(x - 1)$ **b** $y = x(x - 3)$ **c** $y = (x + 1)(x + 5)$

3 Sketch each graph, labelling where the curve crosses the axes.

a $y = x^2 - x - 6$ **b** $y = x^2 - 5x + 4$ **c** $y = x^2 - 4$

d $y = x^2 + 4x$ **e** $y = 9 - x^2$ **f** $y = x^2 + 2x - 3$

4 Sketch the graph of $y = 2x^2 + 5x - 3$, labelling where the curve crosses the axes.

Extend

5 Sketch each graph. Label where the curve crosses the axes and write down the coordinates of the turning point.

a $y = x^2 - 5x + 6$ **b** $y = -x^2 + 7x - 12$ **c** $y = -x^2 + 4x$

6 Sketch the graph of $y = x^2 + 2x + 1$. Label where the curve crosses the axes and write down the equation of the line of symmetry.

Practice questions - Linear inequalities

1 Solve these inequalities.

a $4x > 16$ **b** $5x - 7 \leq 3$ **c** $1 \geq 3x + 4$

d $5 - 2x < 12$ **e** $\frac{x}{2} \geq 5$ **f** $8 < 3 - \frac{x}{3}$

2 Solve these inequalities.

a $\frac{x}{5} < -4$ **b** $10 \geq 2x + 3$ **c** $7 - 3x > -5$

3 Solve

a $2 - 4x \geq 18$ **b** $3 \leq 7x + 10 < 45$ **c** $6 - 2x \geq 4$

d $4x + 17 < 2 - x$ **e** $4 - 5x < -3x$ **f** $-4x \geq 24$

4 Solve these inequalities.

a $3t + 1 < t + 6$ **b** $2(3n - 1) \geq n + 5$

5 Solve.

a $3(2 - x) > 2(4 - x) + 4$ **b** $5(4 - x) > 3(5 - x) + 2$

Extend

6 Find the set of values of x for which $2x + 1 > 11$ and $4x - 2 > 16 - 2x$.

Practice questions – Simultaneous equations

Solve these simultaneous equations.

1 $y = 2x + 1$
$x^2 + y^2 = 10$

2 $y = 6 - x$
$x^2 + y^2 = 20$

3 $y = x - 3$
$x^2 + y^2 = 5$

4 $y = 9 - 2x$
$x^2 + y^2 = 17$

5 $y = 3x - 5$
$y = x^2 - 2x + 1$

6 $y = x - 5$
$y = x^2 - 5x - 12$

7 $y = x + 5$
$x^2 + y^2 = 25$

8 $y = 2x - 1$
$x^2 + xy = 24$

9 $y = 2x$
$y^2 - xy = 8$

10 $2x + y = 11$
$xy = 15$

Extend

11 $x - y = 1$
$x^2 + y^2 = 3$

12 $y - x = 2$
$x^2 + xy = 3$

Practice questions – Quadratic inequalities

1 Find the set of values of x for which $(x + 7)(x - 4) \leq 0$

2 Find the set of values of x for which $x^2 - 4x - 12 \geq 0$

3 Find the set of values of x for which $2x^2 - 7x + 3 < 0$

4 Find the set of values of x for which $4x^2 + 4x - 3 > 0$

5 Find the set of values of x for which $12 + x - x^2 \geq 0$

Extend

Find the set of values which satisfy the following inequalities.

6 $x^2 + x \leq 6$

7 $x(2x - 9) < -10$

8 $6x^2 \geq 15 + x$

Answers – EXPANDING AND SIMPLIFYING EXPRESSIONS

1 **a** $-10pq - 8q^2$

b $40p - 16 - 12p - 27 = 28p - 43$

c $10h - 12h^3 - 22h^2$

d $6b^2$

e $2x^2 + x - 3$

f $6x^2 - x - 2$

g $10x^2 - 31x + 15$

h $12x^2 + 13x - 14$

i $18x^2 + 39xy + 20y^2$

j $x^2 + 10x + 25$

k $4x^2 - 28x + 49$

l $16x^2 - 24xy + 9y^2$

2 **a** $7x(3x - 5) = 21x^2 - 35x$

b $(4x^3 + 12x^2 + 5x - 6)cm^3$

Answers - SURDS

1 a $3\sqrt{5}$

b $4\sqrt{3}$

c $10\sqrt{3}$

d $6\sqrt{2}$

e $15\sqrt{2}$

f $3\sqrt{2}$

g $6\sqrt{7}$

h $5\sqrt{3}$

2a -1

b $10\sqrt{5} - 7\ 3$

a $\frac{\sqrt{5}}{5}$

b $\frac{2\sqrt{7}}{7}$

c $\sqrt{2}$

d $\frac{\sqrt{3}}{3}$

e $\frac{2(4-\sqrt{3})}{13}$

f $\frac{6(5+\sqrt{2})}{23}$

4 $\quad 4\sqrt{3}$

Answers – FACTORISING EXPRESSIONS

1 **a** $5x^2y^2(5 - 2x + 3y)$

b $2(3a - 10bc)(3a + 10bc)$

c $(x - 8)(x + 5)$

d $(x + 7)(x - 4)$

e $(2x + 1)(x + 3)$

f $2(3x - 2)(2x - 5)$

2 **a** $\dfrac{2(x+2)}{x-1}$

b $\dfrac{x+3}{x}$

c $\dfrac{x}{x+5}$

d $\dfrac{2x+3}{3x-2}$

e $\dfrac{2-5x}{2x-3}$

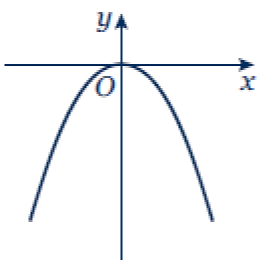
f $\dfrac{4(x+2)}{x-2}$

3 a $\dfrac{8-x}{(x-2)(x+4)}$

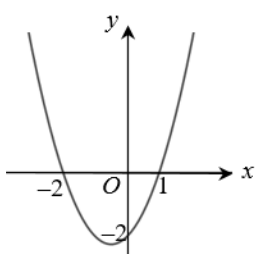
b $x = -8.68\ ,\ 3.68$

Answers – Sketching Quadratics

1



2 **a**



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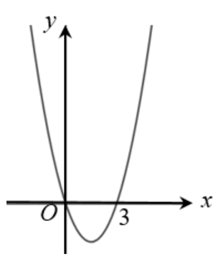

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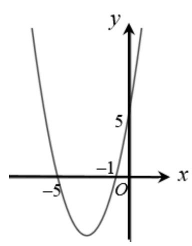
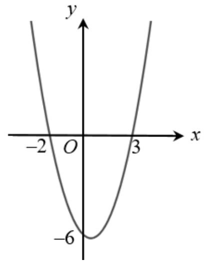


3 **a**



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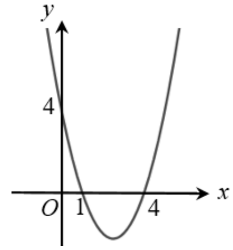

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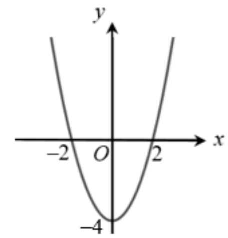

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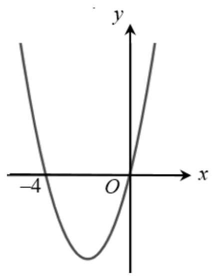

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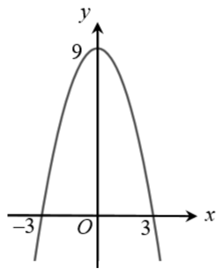

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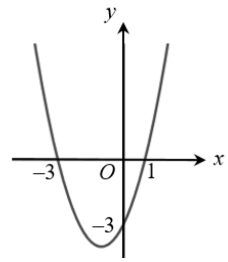

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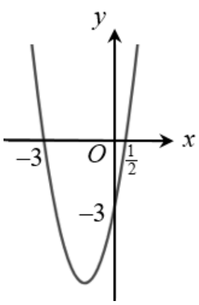
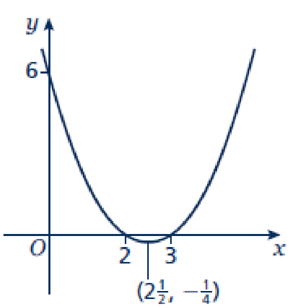


- **5** **a**



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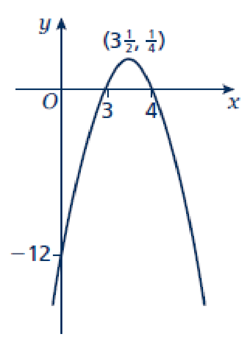

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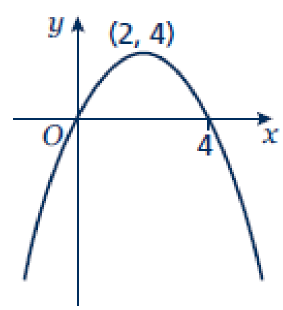

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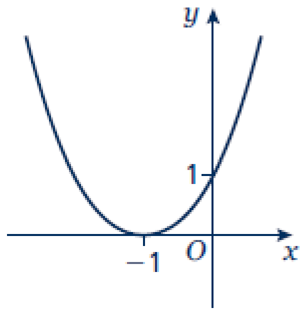

Line of symmetry at $x = -1$.

Answers – Linear inequalities

- **1** **a** $x > 4$ **b** $x \leq 2$ **c** $x \leq -1$
  d $x > -\frac{7}{2}$ **e** $x \geq 10$ **f** $x < -15$
- **2** **a** $x < -20$ **b** $x \leq 3.5$ **c** $x < 4$
- **3** **a** $x \leq -4$ **b** $-1 \leq x < 5$ **c** $x \leq 1$
  d $x < -3$ **e** $x > 2$ **f** $x \leq -6$
- **4** **a** $t < \frac{5}{2}$ **b** $n \geq \frac{7}{5}$
- **5** **a** $x < -6$ **b** $x < \frac{3}{2}$
- **6** $x > 5$ (which also satisfies $x > 3$)

Answers – Simultaneous equations

1 $x = 1, y = 3$
$x = -\frac{9}{5}, y = -\frac{13}{5}$

2 $x = 2, y = 4$
$x = 4, y = 2$

3 $x = 1, y = -2$
$x = 2, y = -1$

4 $x = 4, y = 1$
$x = \frac{16}{5}, y = \frac{13}{5}$

5 $x = 3, y = 4$
$x = 2, y = 1$

6 $x = 7, y = 2$
$x = -1, y = -6$

7 $x = 0, y = 5$
$x = -5, y = 0$

8 $x = -\frac{8}{3}, y = -\frac{19}{3}$
$x = 3, y = 5$

9 $x = -2, y = -4$
$x = 2, y = 4$

10 $x = \frac{5}{2}, y = 6$
$x = 3, y = 5$

11 $x = \frac{1+\sqrt{5}}{2}, y = \frac{-1+\sqrt{5}}{2}$
$x = \frac{1-\sqrt{5}}{2}, y = \frac{-1-\sqrt{5}}{2}$

12 $x = \frac{-1+\sqrt{7}}{2}, y = \frac{3+\sqrt{7}}{2}$
$x = \frac{-1-\sqrt{7}}{2}, y = \frac{3-\sqrt{7}}{2}$

Answers – Quadratic inequalities

- **1** $-7 \le x \le 4$
- **2** $x \le -2$ or $x \ge 6$
- **3** $\frac{1}{2} < x < 3$
- **4** $x < -\frac{3}{2}$ or $x > \frac{1}{2}$
- **5** $-3 \le x \le 4$
- **6** $-3 \le x \le 2$

7 $2 < x < 2\frac{1}{2}$

8 $x \le -\frac{3}{2}$ or $x \ge \frac{5}{3}$

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NOTES and EXAMPLES BOOKLET

In order to succeed on this course it is imperative that you start in September being proficient at a number of key skills from GCSE Mathematics. In order to ensure that this is the case, this booklet consists of notes and examples of some high level GCSE questions, which you will need read through thoroughly. Once you have read through this booklet you will need to complete **the practice question booklet** (attached)

The expectation is that you are fully competent in applying these skills and will have the necessary perseverance to complete all the practice questions attached.

During your **first double A Level lesson** week beginning **Monday 11th September**, you will take a ***60 minute test*** to assess the topics covered in this booklet.

All students who do not complete the question booklet fully or who perform poorly in the test will then need to consider carefully whether they have the aptitude required to succeed on the demanding Mathematics A Level course.

Expanding brackets and simplifying expressions

Key points

- When you expand one set of brackets you must multiply everything inside the bracket by what is outside.
- When you expand two linear expressions, each with two terms of the form $ax + b$, where $a \neq 0$ and $b \neq 0$, you create four terms. Two of these can usually be simplified by collecting like terms.

Examples

Example 1 Expand $4(3x - 2)$

$4(3x - 2) = 12x - 8$	Multiply everything inside the bracket by the 4 outside the bracket

Example 2 Expand and simplify $3(x + 5) - 4(2x + 3)$

$3(x + 5) - 4(2x + 3)$ $= 3x + 15 - 8x - 12$	**1** Expand each set of brackets separately by multiplying $(x + 5)$ by 3 and $(2x + 3)$ by -4
$= 3 - 5x$	**2** Simplify by collecting like terms: $3x - 8x = -5x$ and $15 - 12 = 3$

Example 3 Expand and simplify $(x + 3)(x + 2)$

$(x + 3)(x + 2)$ $= x(x + 2) + 3(x + 2)$ $= x^2 + 2x + 3x + 6$	**1** Expand the brackets by multiplying $(x + 2)$ by x and $(x + 2)$ by 3
$= x^2 + 5x + 6$	**2** Simplify by collecting like terms: $2x + 3x = 5x$

Example 4 Expand and simplify $(x - 5)(2x + 3)$

$(x - 5)(2x + 3)$ $= x(2x + 3) - 5(2x + 3)$ $= 2x^2 + 3x - 10x - 15$	**1** Expand the brackets by multiplying $(2x + 3)$ by x and $(2x + 3)$ by -5
$= 2x^2 - 7x - 15$	**2** Simplify by collecting like terms: $3x - 10x = -7x$

Surds and rationalising the denominator

Key points

- A surd is the square root of a number that is not a square number, for example $\sqrt{2}, \sqrt{3}, \sqrt{5},$ etc.
- Surds can be used to give the exact value for an answer.
- $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$
- $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$
- To rationalise the denominator means to remove the surd from the denominator of a fraction.
- To rationalise $\frac{a}{\sqrt{b}}$ you multiply the numerator and denominator by the surd $\sqrt{b}$
- To rationalise $\frac{a}{b+\sqrt{c}}$ you multiply the numerator and denominator by $b-\sqrt{c}$

Examples

Example 1 Simplify $\sqrt{50}$

$\sqrt{50} = \sqrt{25 \times 2}$	**1** Choose two numbers that are factors of 50. One of the factors must be a square number
$= \sqrt{25} \times \sqrt{2}$	**2** Use the rule $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$
$= 5 \times \sqrt{2}$	**3** Use $\sqrt{25} = 5$
$= 5\sqrt{2}$	

Example 2 Simplify $\sqrt{147} - 2\sqrt{12}$

$\sqrt{147} - 2\sqrt{12}$ $= \sqrt{49 \times 3} - 2\sqrt{4 \times 3}$	**1** Simplify $\sqrt{147}$ and $2\sqrt{12}$. Choose two numbers that are factors of 147 and two numbers that are factors of 12. One of each pair of factors must be a square number
$= \sqrt{49} \times \sqrt{3} - 2\sqrt{4} \times \sqrt{3}$	**2** Use the rule $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$
$= 7 \times \sqrt{3} - 2 \times 2 \times \sqrt{3}$	**3** Use $\sqrt{49} = 7$ and $\sqrt{4} = 2$
$= 7\sqrt{3} - 4\sqrt{3}$ $= 3\sqrt{3}$	**4** Collect like terms

Example 3 Simplify $\left(\sqrt{7} + \sqrt{2}\right)\left(\sqrt{7} - \sqrt{2}\right)$

$\left(\sqrt{7} + \sqrt{2}\right)\left(\sqrt{7} - \sqrt{2}\right)$ $= \sqrt{49} - \sqrt{7}\sqrt{2} + \sqrt{2}\sqrt{7} - \sqrt{4}$	**1** Expand the brackets. A common mistake here is to write $\left(\sqrt{7}\right)^2 = 49$
$= 7 - 2$	

$= 5$	**2** Collect like terms: $-\sqrt{7}\sqrt{2}+\sqrt{2}\sqrt{7}$ $= -\sqrt{7}\sqrt{2}+\sqrt{7}\sqrt{2} = 0$

Example 4 Rationalise $\frac{1}{\sqrt{3}}$

$\frac{1}{\sqrt{3}} = \frac{1}{\sqrt{3}}\times\frac{\sqrt{3}}{\sqrt{3}}$	**1** Multiply the numerator and denominator by $\sqrt{3}$
$= \frac{1\times\sqrt{3}}{\sqrt{9}}$	**2** Use $\sqrt{9}=3$
$= \frac{\sqrt{3}}{3}$	

Example 5 Rationalise and simplify $\frac{\sqrt{2}}{\sqrt{12}}$

$\frac{\sqrt{2}}{\sqrt{12}} = \frac{\sqrt{2}}{\sqrt{12}}\times\frac{\sqrt{12}}{\sqrt{12}}$	**1** Multiply the numerator and denominator by $\sqrt{12}$
$= \frac{\sqrt{2}\times\sqrt{4\times3}}{12}$	**2** Simplify $\sqrt{12}$ in the numerator. Choose two numbers that are factors of 12. One of the factors must be a square number
	3 Use the rule $\sqrt{ab}=\sqrt{a}\times\sqrt{b}$
$= \frac{2\sqrt{2}\sqrt{3}}{12}$	**4** Use $\sqrt{4}=2$
$= \frac{\sqrt{2}\sqrt{3}}{6}$	**5** Simplify the fraction: $\frac{2}{12}$ simplifies to $\frac{1}{6}$

Example 6 Rationalise and simplify $\frac{3}{2+\sqrt{5}}$

$\frac{3}{2+\sqrt{5}} = \frac{3}{2+\sqrt{5}} \times \frac{2-\sqrt{5}}{2-\sqrt{5}}$	**1** Multiply the numerator and denominator by $2-\sqrt{5}$
$= \frac{3\left(2-\sqrt{5}\right)}{\left(2+\sqrt{5}\right)\left(2-\sqrt{5}\right)}$	**2** Expand the brackets
$= \frac{6-3\sqrt{5}}{4+2\sqrt{5}-2\sqrt{5}-5}$	**3** Simplify the fraction
$= \frac{6-3\sqrt{5}}{-1}$	**4** Divide the numerator by −1 Remember to change the sign of all terms when dividing by −1
$= 3\sqrt{5}-6$	

Rules of indices

Key points

- $a^m \times a^n = a^{m+n}$
- $\frac{a^m}{a^n} = a^{m-n}$
- $(a^m)^n = a^{mn}$
- $a^0 = 1$
- $a^{\frac{1}{n}} = \sqrt[n]{a}$ i.e. the nth root of a
- $a^{\frac{m}{n}} = \sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m$
- $a^{-m} = \frac{1}{a^m}$
- The square root of a number produces two solutions, e.g. $\sqrt{16} = \pm 4$.

Examples

Example 1 Evaluate 10^0

$10^0 = 1$	Any value raised to the power of zero is equal to 1

Example 2 Evaluate $9^{\frac{1}{2}}$

$9^{\frac{1}{2}} = \sqrt{9}$ $= 3$	Use the rule $a^{\frac{1}{n}} = \sqrt[n]{a}$

Example 3 Evaluate $27^{\frac{2}{3}}$

$27^{\frac{2}{3}} = \left(\sqrt[3]{27}\right)^2$ $= 3^2$ $= 9$	**1** Use the rule $a^{\frac{m}{n}} = \left(\sqrt[n]{a}\right)^m$ **2** Use $\sqrt[3]{27} = 3$

Example 4 Evaluate 4^{-2}

$4^{-2} = \dfrac{1}{4^2}$ $= \dfrac{1}{16}$	**1** Use the rule $a^{-m} = \dfrac{1}{a^m}$ **2** Use $4^2 = 16$

Example 5 Simplify $\dfrac{6x^5}{2x^2}$

$\dfrac{6x^5}{2x^2} = 3x^3$	$6 \div 2 = 3$ and use the rule $\dfrac{a^m}{a^n} = a^{m-n}$ to give $\dfrac{x^5}{x^2} = x^{5-2} = x^3$

Example 6 Simplify $\dfrac{x^3 \times x^5}{x^4}$

$\dfrac{x^3 \times x^5}{x^4} = \dfrac{x^{3+5}}{x^4} = \dfrac{x^8}{x^4}$ $= x^{8-4} = x^4$	**1** Use the rule $a^m \times a^n = a^{m+n}$ **2** Use the rule $\dfrac{a^m}{a^n} = a^{m-n}$

Example 7 Write $\dfrac{1}{3x}$ as a single power of x

$\dfrac{1}{3x} = \dfrac{1}{3}x^{-1}$	Use the rule $\dfrac{1}{a^m} = a^{-m}$, note that the fraction $\dfrac{1}{3}$ remains unchanged

Example 8 Write $\dfrac{4}{\sqrt{x}}$ as a single power of x

$\frac{4}{\sqrt{x}} = \frac{4}{x^{\frac{1}{2}}}$ $= 4x^{-\frac{1}{2}}$	**1** Use the rule $a^{\frac{1}{n}} = \sqrt[n]{a}$ **2** Use the rule $\frac{1}{a^m} = a^{-m}$

Factorising expressions

Key points

- Factorising an expression is the opposite of expanding the brackets.
- A quadratic expression is in the form $ax^2 + bx + c$, where $a \neq 0$.
- To factorise a quadratic equation find two numbers whose sum is b and whose product is ac.
- An expression in the form $x^2 - y^2$ is called the difference of two squares. It factorises to $(x - y)(x + y)$.

Examples

Example 1 Factorise $15x^2y^3 + 9x^4y$

$15x^2y^3 + 9x^4y = 3x^2y(5y^2 + 3x^2)$	The highest common factor is $3x^2y$. So take $3x^2y$ outside the brackets and then divide each term by $3x^2y$ to find the terms in the brackets

Example 2 Factorise $4x^2 - 25y^2$

$4x^2 - 25y^2 = (2x + 5y)(2x - 5y)$	This is the difference of two squares as the two terms can be written as $(2x)^2$ and $(5y)^2$

Example 3 Factorise $x^2 + 3x - 10$

$b = 3$, $ac = -10$	**1** Work out the two factors of $ac = -10$ which add to give $b = 3$ (5 and −2)
So $x^2 + 3x - 10 = x^2 + 5x - 2x - 10$	**2** Rewrite the b term ($3x$) using these two factors
$= x(x + 5) - 2(x + 5)$	**3** Factorise the first two terms and the last two terms
$= (x + 5)(x - 2)$	**4** $(x + 5)$ is a factor of both terms

Example 4 Factorise $6x^2 - 11x - 10$

$b = -11$, $ac = -60$	**1** Work out the two factors of $ac = -60$ which add to give $b = -11$ (−15 and 4)
So $6x^2 - 11x - 10 = 6x^2 - 15x + 4x - 10$	**2** Rewrite the b term ($-11x$) using these two factors
$= 3x(2x - 5) + 2(2x - 5)$	**3** Factorise the first two terms and the last two terms
$= (2x - 5)(3x + 2)$	**4** $(2x - 5)$ is a factor of both terms

Example 5 Simplify $\dfrac{x^2-4x-21}{2x^2+9x+9}$

$\dfrac{x^2-4x-21}{2x^2+9x+9}$	**1** Factorise the numerator and the denominator
For the numerator: $b = -4$, $ac = -21$	**2** Work out the two factors of $ac = -21$ which add to give $b = -4$ (-7 and 3)
So $x^2 - 4x - 21 = x^2 - 7x + 3x - 21$	**3** Rewrite the b term ($-4x$) using these two factors
$= x(x - 7) + 3(x - 7)$	**4** Factorise the first two terms and the last two terms
$= (x - 7)(x + 3)$	**5** $(x - 7)$ is a factor of both terms
For the denominator: $b = 9$, $ac = 18$	**6** Work out the two factors of $ac = 18$ which add to give $b = 9$ (6 and 3)
So $2x^2 + 9x + 9 = 2x^2 + 6x + 3x + 9$	**7** Rewrite the b term ($9x$) using these two factors
$= 2x(x + 3) + 3(x + 3)$	**8** Factorise the first two terms and the last two terms
$= (x + 3)(2x + 3)$	**9** $(x + 3)$ is a factor of both terms
So $\dfrac{x^2-4x-21}{2x^2+9x+9} = \dfrac{(x-7)(x+3)}{(x+3)(2x+3)}$ $= \dfrac{x-7}{2x+3}$	**10** $(x + 3)$ is a factor of both the numerator and denominator so cancels out as a value divided by itself is 1

Solving quadratic equations by factorisation

Key points

- A quadratic equation is an equation in the form $ax^2 + bx + c = 0$ where $a \neq 0$.
- To factorise a quadratic equation find two numbers whose sum is b and whose products is ac.
- When the product of two numbers is 0, then at least one of the numbers must be 0.
- If a quadratic can be solved it will have two solutions (these may be equal).

Examples

Example 1 Solve $5x^2 = 15x$

$5x^2 = 15x$ $5x^2 - 15x = 0$	**1** Rearrange the equation so that all of the terms are on one side of the equation and it is equal to zero. Do not divide both sides by x as this would lose the solution $x = 0$.
$5x(x - 3) = 0$	**2** Factorise the quadratic equation. $5x$ is a common factor.
So $5x = 0$ or $(x - 3) = 0$	**3** When two values multiply to make zero, at least one of the values must be zero.
Therefore $x = 0$ or $x = 3$	**4** Solve these two equations.

Example 2 Solve $x^2 + 7x + 12 = 0$

$x^2 + 7x + 12 = 0$ $b = 7$, $ac = 12$	**1** Factorise the quadratic equation. Work out the two factors of $ac = 12$ which add to give you $b = 7$. (4 and 3)
$x^2 + 4x + 3x + 12 = 0$	**2** Rewrite the b term ($7x$) using these two factors.
$x(x + 4) + 3(x + 4) = 0$	**3** Factorise the first two terms and the last two terms.
$(x + 4)(x + 3) = 0$	**4** $(x + 4)$ is a factor of both terms.
So $(x + 4) = 0$ or $(x + 3) = 0$	**5** When two values multiply to make zero, at least one of the values must be zero.
Therefore $x = -4$ or $x = -3$	**6** Solve these two equations.

Example 3 Solve $9x^2 - 16 = 0$

$9x^2 - 16 = 0$ $(3x + 4)(3x - 4) = 0$	**1** Factorise the quadratic equation. This is the difference of two squares as the two terms are $(3x)^2$ and $(4)^2$.
So $(3x + 4) = 0$ or $(3x - 4) = 0$	**2** When two values multiply to make zero, at least one of the values must be zero.
$x = -\frac{4}{3}$ or $x = \frac{4}{3}$	**3** Solve these two equations.

Example 4 Solve $2x^2 - 5x - 12 = 0$

$b = -5$, $ac = -24$	**1** Factorise the quadratic equation. Work out the two factors of $ac = -24$ which add to give you $b = -5$. (-8 and 3)
So $2x^2 - 8x + 3x - 12 = 0$	**2** Rewrite the b term ($-5x$) using these two factors.
$2x(x - 4) + 3(x - 4) = 0$	**3** Factorise the first two terms and the last two terms.
$(x - 4)(2x + 3) = 0$	**4** $(x - 4)$ is a factor of both terms.
So $(x - 4) = 0$ or $(2x + 3) = 0$	**5** When two values multiply to make zero, at least one of the values must be zero.
$x = 4$ or $x = -\frac{3}{2}$	**6** Solve these two equations.

Solving quadratic equations by completing the square

Key points

- Completing the square lets you write a quadratic equation in the form $p(x + q)^2 + r = 0$.

Examples

Example 5 Solve $x^2 + 6x + 4 = 0$. Give your solutions in surd form.

$x^2 + 6x + 4 = 0$	**1** Write $x^2 + bx + c = 0$ in the form $\left(x + \frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2 + c = 0$
$(x + 3)^2 - 9 + 4 = 0$	
$(x + 3)^2 - 5 = 0$	**2** Simplify.
$(x + 3)^2 = 5$	**3** Rearrange the equation to work out x. First, add 5 to both sides.
$x + 3 = \pm\sqrt{5}$	**4** Square root both sides. Remember that the square root of a value gives two answers.
$x = \pm\sqrt{5} - 3$	**5** Subtract 3 from both sides to solve the equation.
So $x = -\sqrt{5} - 3$ or $x = \sqrt{5} - 3$	**6** Write down both solutions.

Example 6 Solve $2x^2 - 7x + 4 = 0$. Give your solutions in surd form.

$2x^2 - 7x + 4 = 0$	**1** Before completing the square write $ax^2 + bx + c$ in the form $a\left(x^2 + \frac{b}{a}x\right) + c$
$2\left(x^2 - \frac{7}{2}x\right) + 4 = 0$	
$2\left[\left(x - \frac{7}{4}\right)^2 - \left(\frac{7}{4}\right)^2\right] + 4 = 0$	**2** Now complete the square by writing $x^2 - \frac{7}{2}x$ in the form $\left(x + \frac{b}{2a}\right)^2 - \left(\frac{b}{2a}\right)^2$
$2\left(x - \frac{7}{4}\right)^2 - \frac{49}{8} + 4 = 0$	**3** Expand the square brackets.
$2\left(x - \frac{7}{4}\right)^2 - \frac{17}{8} = 0$	**4** Simplify.
	(continued on next page)
$2\left(x - \frac{7}{4}\right)^2 = \frac{17}{8}$	**5** Rearrange the equation to work out x. First, add $\frac{17}{8}$ to both sides.
$\left(x - \frac{7}{4}\right)^2 = \frac{17}{16}$	**6** Divide both sides by 2.
$x - \frac{7}{4} = \pm\frac{\sqrt{17}}{4}$	**7** Square root both sides. Remember that the square root of a value gives two answers.
$x = \pm\frac{\sqrt{17}}{4} + \frac{7}{4}$	**8** Add $\frac{7}{4}$ to both sides.
So $x = \frac{7}{4} - \frac{\sqrt{17}}{4}$ or $x = \frac{7}{4} + \frac{\sqrt{17}}{4}$	**9** Write down both the solutions.

Solving quadratic equations by using the formula

Key points

- Any quadratic equation of the form $ax^2 + bx + c = 0$ can be solved using the formula $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
- If $b^2 - 4ac$ is negative then the quadratic equation does not have any real solutions.
- It is useful to write down the formula before substituting the values for a, b and c.

Examples

Example 7 Solve $x^2 + 6x + 4 = 0$. Give your solutions in surd form.

$a = 1$, $b = 6$, $c = 4$ $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	**1** Identify a, b and c and write down the formula. Remember that $-b \pm \sqrt{b^2 - 4ac}$ is all over $2a$, not just part of it.
$x = \dfrac{-6 \pm \sqrt{6^2 - 4(1)(4)}}{2(1)}$	**2** Substitute $a = 1$, $b = 6$, $c = 4$ into the formula.
$x = \dfrac{-6 \pm \sqrt{20}}{2}$	**3** Simplify. The denominator is 2, but this is only because $a = 1$. The denominator will not always be 2.
$x = \dfrac{-6 \pm 2\sqrt{5}}{2}$	**4** Simplify $\sqrt{20}$. $\sqrt{20} = \sqrt{4 \times 5} = \sqrt{4} \times \sqrt{5} = 2\sqrt{5}$
$x = -3 \pm \sqrt{5}$	**5** Simplify by dividing numerator and denominator by 2.
So $x = -3 - \sqrt{5}$ or $x = \sqrt{5} - 3$	**6** Write down both the solutions.

Example 8 Solve $3x^2 - 7x - 2 = 0$. Give your solutions in surd form.

$a = 3, b = -7, c = -2$ $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	**1** Identify a, b and c, making sure you get the signs right and write down the formula. Remember that $-b \pm \sqrt{b^2 - 4ac}$ is all over $2a$, not just part of it.
$x = \dfrac{-(-7) \pm \sqrt{(-7)^2 - 4(3)(-2)}}{2(3)}$	**2** Substitute $a = 3$, $b = -7$, $c = -2$ into the formula.
$x = \dfrac{7 \pm \sqrt{73}}{6}$ So $x = \dfrac{7 - \sqrt{73}}{6}$ or	**3** Simplify. The denominator is 6 when $a = 3$. A common mistake is to always write a denominator of 2.
$x = \dfrac{7 + \sqrt{73}}{6}$	**4** Write down both the solutions.

Linear inequalities

Key points

- Solving linear inequalities uses similar methods to those for solving linear equations.
- When you multiply or divide an inequality by a negative number you need to reverse the inequality sign, e.g. < becomes >.

Examples

Example 1 Solve $-8 \le 4x < 16$

$-8 \le 4x < 16$ $-2 \le x < 4$	Divide all three terms by 4.

Example 2 Solve $4 \le 5x < 10$

$4 \le 5x < 10$ $\frac{4}{5} \le x < 2$	Divide all three terms by 5.

Example 3 Solve $2x - 5 < 7$

$2x - 5 < 7$ $2x < 12$ $x < 6$	1 Add 5 to both sides. 2 Divide both sides by 2.

Example 4 Solve $2 - 5x \geq -8$

$2 - 5x \geq -8$ $-5x \geq -10$ $x \leq 2$	1 Subtract 2 from both sides. 2 Divide both sides by -5. Remember to reverse the inequality when dividing by a negative number.

Example 5 Solve $4(x - 2) > 3(9 - x)$

$4(x - 2) > 3(9 - x)$ $4x - 8 > 27 - 3x$ $7x - 8 > 27$ $7x > 35$ $x > 5$	1 Expand the brackets. 2 Add $3x$ to both sides. 3 Add 8 to both sides. 4 Divide both sides by 7.

Solving linear and quadratic simultaneous equations

Key points

- Make one of the unknowns the subject of the linear equation (rearranging where necessary).
- Use the linear equation to substitute into the quadratic equation.
- There are usually two pairs of solutions.

Examples

Example 1 Solve the simultaneous equations $y = x + 1$ and $x^2 + y^2 = 13$

$x^2 + (x + 1)^2 = 13$	1 Substitute $x + 1$ for y into the second equation.
$x^2 + x^2 + x + x + 1 = 13$ $2x^2 + 2x + 1 = 13$	2 Expand the brackets and simplify.
$2x^2 + 2x - 12 = 0$ $(2x - 4)(x + 3) = 0$	3 Factorise the quadratic equation.
So $x = 2$ or $x = -3$	4 Work out the values of x.

Using $y = x + 1$ When $x = 2$, $y = 2 + 1 = 3$ When $x = -3$, $y = -3 + 1 = -2$ So the solutions are $x = 2$, $y = 3$ and $x = -3$, $y = -2$ Check: equation 1: $3 = 2 + 1$ YES and $-2 = -3 + 1$ YES equation 2: $2^2 + 3^2 = 13$ YES and $(-3)^2 + (-2)^2 = 13$ YES	5 To find the value of y, substitute both values of x into one of the original equations. 6 Substitute both pairs of values of x and y into both equations to check your answers.

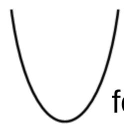
Example 2 Solve $2x + 3y = 5$ and $2y^2 + xy = 12$ simultaneously.

$x = \frac{5-3y}{2}$	1 Rearrange the first equation.
$2y^2 + \left(\frac{5-3y}{2}\right)y = 12$	2 Substitute $\frac{5-3y}{2}$ for x into the second equation. Notice how it is easier to substitute for x than for y.
$2y^2 + \frac{5y-3y^2}{2} = 12$ $4y^2 + 5y - 3y^2 = 24$ $y^2 + 5y - 24 = 0$	3 Expand the brackets and simplify.
$(y + 8)(y - 3) = 0$	4 Factorise the quadratic equation.
So $y = -8$ or $y = 3$	5 Work out the values of y.
Using $2x + 3y = 5$ When $y = -8$, $2x + 3 \times (-8) = 5$, $x = 14.5$ When $y = 3$, $2x + 3 \times 3 = 5$, $x = -2$	6 To find the value of x, substitute both values of y into one of the original equations.
So the solutions are $x = 14.5$, $y = -8$ and $x = -2$, $y = 3$ Check: equation 1: $2 \times 14.5 + 3 \times (-8) = 5$ YES and $2 \times (-2) + 3 \times 3 = 5$ YES equation 2: $2\times(-8)^2 + 14.5\times(-8) = 12$ YES and $2 \times (3)^2 + (-2) \times 3 = 12$ YES	7 Substitute both pairs of values of x and y into both equations to check your answers.

<u>Sketching quadratic graphs</u>

Key points

- The graph of the quadratic function $y = ax^2 + bx + c$, where $a \neq 0$, is a curve called a parabola.
- Parabolas have a line of symmetry and a shape as shown.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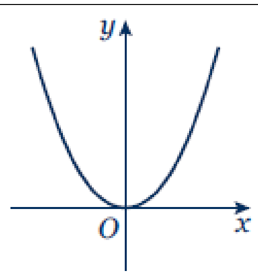
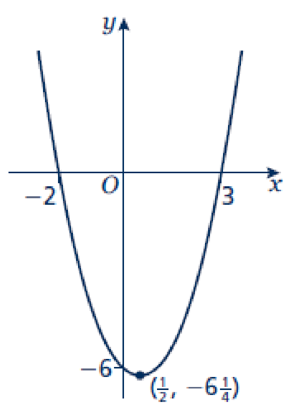
- To sketch the graph of a function, find the points where the graph intersects the axes.
- To find where the curve intersects the y-axis substitute $x = 0$ into the function.
- To find where the curve intersects the x-axis substitute $y = 0$ into the function.
- At the turning points of a graph the gradient of the curve is 0 and any tangents to the curve at these points are horizontal.
- To find the coordinates of the maximum or minimum point (turning points) of a quadratic curve (parabola) you can use the completed square form of the function.

<u>Examples</u>

Example 1 Sketch the graph of $y = x^2$.

	The graph of $y = x^2$ is a parabola. When $x = 0$, $y = 0$. $a = 1$ which is greater than zero, so the graph has the shape: 

Example 2 Sketch the graph of $y = x^2 - x - 6$.

When $x = 0$, $y = 0^2 - 0 - 6 = -6$ So the graph intersects the y-axis at $(0, -6)$	1 Find where the graph intersects the y-axis by substituting $x = 0$.
When $y = 0$, $x^2 - x - 6 = 0$	2 Find where the graph intersects the x-axis by substituting $y = 0$.
$(x + 2)(x - 3) = 0$	3 Solve the equation by factorising.
$x = -2$ or $x = 3$	4 Solve $(x + 2) = 0$ and $(x - 3) = 0$.
So, the graph intersects the x-axis at $(-2, 0)$ and $(3, 0)$	5 $a = 1$ which is greater than zero, so the graph has the shape: 
$x^2 - x - 6 = \left(x - \frac{1}{2}\right)^2 - \frac{1}{4} - 6$ $= \left(x - \frac{1}{2}\right)^2 - \frac{25}{4}$	6 To find the turning point, complete the square.
When $\left(x - \frac{1}{2}\right)^2 = 0$, $x = \frac{1}{2}$ and $y = -\frac{25}{4}$, so the turning point is at the point $\left(\frac{1}{2}, -\frac{25}{4}\right)$	7 The turning point is the minimum value for this expression and occurs when the term in the bracket is equal to zero.
 	

<u>Linear inequalities</u>

Key points

- Solving linear inequalities uses similar methods to those for solving linear equations.
- When you multiply or divide an inequality by a negative number you need to reverse the inequality sign, e.g. $<$ becomes $>$.

<u>Examples</u>

Example 1 Solve $-8 \le 4x < 16$

$-8 \le 4x < 16$ $-2 \le x < 4$	Divide all three terms by 4.

Example 2 Solve $4 \le 5x < 10$

$4 \le 5x < 10$ $\frac{4}{5} \le x < 2$	Divide all three terms by 5.

Example 3 Solve $2x - 5 < 7$

$2x - 5 < 7$ $2x < 12$ $x < 6$	1 Add 5 to both sides. 2 Divide both sides by 2.

Example 4 Solve $2 - 5x \ge -8$

$2 - 5x \ge -8$ $-5x \ge -10$ $x \le 2$	1 Subtract 2 from both sides. 2 Divide both sides by -5. Remember to reverse the inequality when dividing by a negative number.

Example 5 Solve $4(x - 2) > 3(9 - x)$

$4(x - 2) > 3(9 - x)$ $4x - 8 > 27 - 3x$ $7x - 8 > 27$ $7x > 35$ $x > 5$	1 Expand the brackets. 2 Add $3x$ to both sides. 3 Add 8 to both sides. 4 Divide both sides by 7.

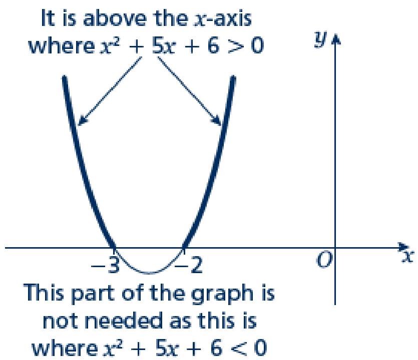
<u>Quadratic inequalities</u>

Key points

- First replace the inequality sign by = and solve the quadratic equation.
- Sketch the graph of the quadratic function.
- Use the graph to find the values which satisfy the quadratic inequality.

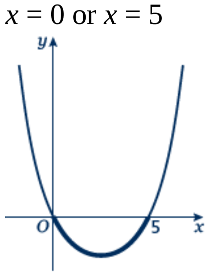
<u>Examples</u>

Example 1 Find the set of values of x which satisfy $x^2 + 5x + 6 > 0$

<table>
<tr><td>$x^2 + 5x + 6 = 0$
$(x + 3)(x + 2) = 0$
$x = -3$ or $x = -2$</td><td>1 Solve the quadratic equation by factorising.</td></tr>
<tr><td>
</td><td>2 Sketch the graph of $y = (x + 3)(x + 2)$

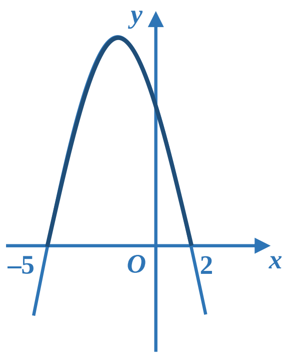
3 Identify on the graph where $x^2 + 5x + 6 > 0$, i.e. where $y > 0$</td></tr>
<tr><td>$x < -3$ or $x > -2$</td><td>4 Write down the values which satisfy the inequality $x^2 + 5x + 6 > 0$</td></tr>
</table>

Example 2 Find the set of values of x which satisfy $x^2 - 5x \leq 0$

<table>
<tr><td>$x^2 - 5x = 0$
$x(x - 5) = 0$
$x = 0$ or $x = 5$</td><td>1 Solve the quadratic equation by factorising.</td></tr>
<tr><td>
</td><td>2 Sketch the graph of $y = x(x - 5)$

3 Identify on the graph where $x^2 - 5x \leq 0$, i.e. where $y \leq 0$</td></tr>
<tr><td>$0 \leq x \leq 5$</td><td>4 Write down the values which satisfy the inequality $x^2 - 5x \leq 0$</td></tr>
</table>

Example 3 Find the set of values of x which satisfy $-x^2 - 3x + 10 \geq 0$

$-x^2 - 3x + 10 = 0$ $(-x + 2)(x + 5) = 0$ $x = 2$ or $x = -5$	1 Solve the quadratic equation by factorising.
	2 Sketch the graph of $y = (-x + 2)(x + 5) = 0$ 3 Identify on the graph where $-x^2 - 3x + 10 \geq 0$, i.e. where $y \geq 0$
$-5 \leq x \leq 2$	3 Write down the values which satisfy the inequality $-x^2 - 3x + 10 \geq 0$