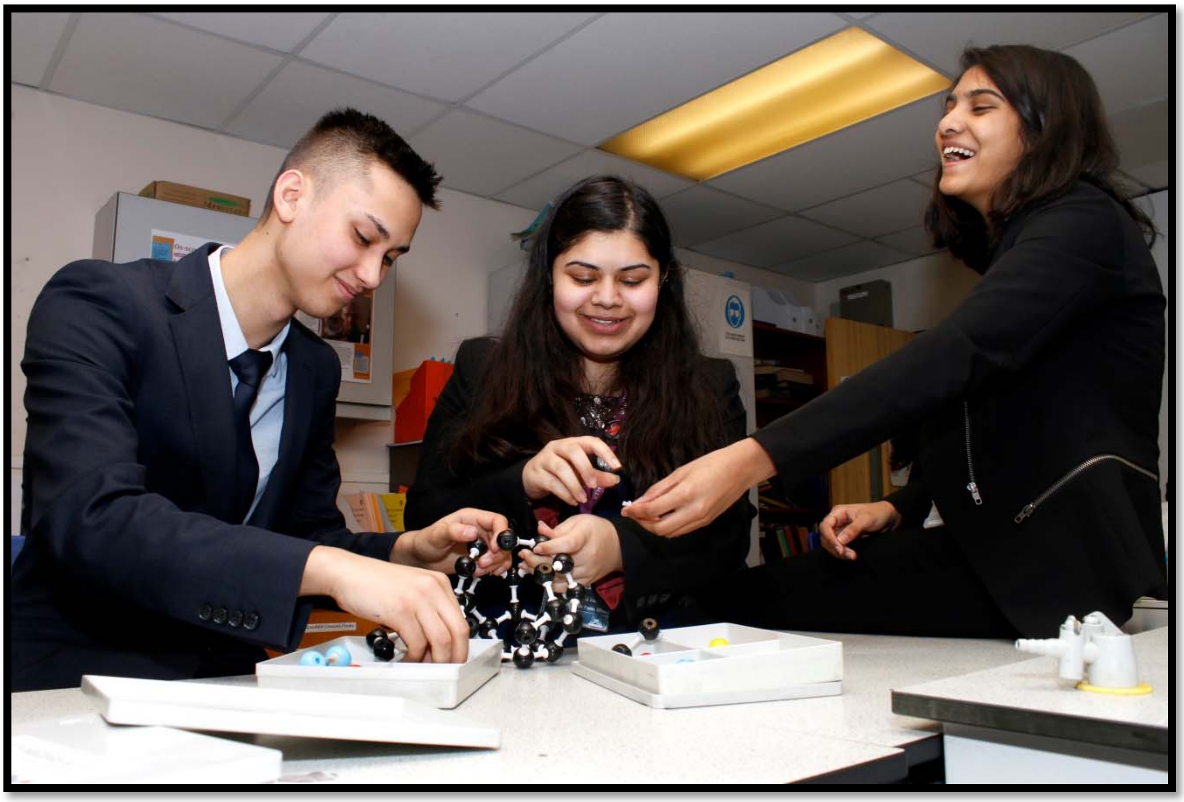
The Heathland School

A Mathematics and Science College



SIXTH FORM

INDUCTION TASKS




Dear student,

Congratulations on your enrolment for the Sixth Form at The Heathland School.

The leap from GCSE to Post 16 study is significant and it is essential that you make a strong and committed start to your courses in September.

In order to help you do this, we have asked departments to prepare some preliminary work for you to start before your first lessons begin. There are tasks to complete for each A Level or BTEC subject you are going to study in Year 12. Teachers will refer to these tasks during the first two weeks of study.

I would also ask you to view the specification for each subject by viewing the curriculum section on the school website.

The best of luck with your Sixth Form studies – we look forward to seeing you make good progress during Year 12 and beyond.

Personalised Checklists (PLCS)

A PLC is a Personalised Learning Checklist. It is an organised list of topics that you will study in your chosen subjects taken from the syllabus. It also provides an opportunity for you to reflect on your progress in your subjects.

MyPLC (https://www.my-plc.co.uk/register/) has a large bank of subject and exam board specific information. Sign up as a student and join the Sixth Form Students class by entering the code **ab4870.**

You will then have access to all the available PLC's for your subject and exam board. This will:

1. Show you all the topics you will be studying for your subjects
2. Allow you to rate your level of understanding for each topic as you study them
3. Help you direct your revision to make it specific, focused and individual to you; ensuring your revision is an effective use of time and energy

Previous students have said:

"PLC's help me see in advance what we will be learning so I can do some additional reading before the lesson"

"Using the PLC has helped me to focus my revision on areas I need to improve"

"It has been really helpful when Topic tests come up. I know specifically what to revise"

The Heathland School Mathematics Department

A Level Course Preparation Questions 2019

In order to succeed on this course it is imperative that you start in September being proficient at a number of key skills from GCSE Mathematics. In order to ensure that this is the case, this question booklet consists of notes, examples and some high level GCSE questions, which you will need to complete fully by ***Monday 9th September 2019.***

Numerical answers are given at the end of each section so that you are able to tell if you have completed the questions correctly. Teachers will be checking for full working out which shows complete understanding for all questions when they collect the booklets in on Monday 9th September. The expectation is that you will have shown the necessary perseverance to complete all questions fully. If there are any gaps at all, the booklet will be considered to be incomplete.

During your first A Level lesson week beginning 9th September, you will take a ***40 minute test*** to assess the topics covered in this booklet. Calculators may be used both in completing this booklet and the test but method must be shown.

All students who do not complete the booklet fully or who perform poorly in the test will then need to consider carefully whether or not they have the aptitude required to succeed on the demanding Mathematics A Level course.

<u>Expanding brackets and simplifying expressions</u>

Key points

- When you expand one set of brackets you must multiply everything inside the bracket by what is outside.
- When you expand two linear expressions, each with two terms of the form $ax + b$, where $a \neq 0$ and $b \neq 0$, you create four terms. Two of these can usually be simplified by collecting like terms.

Examples

Example 1 Expand $4(3x - 2)$

$4(3x - 2) = 12x - 8$	Multiply everything inside the bracket by the 4 outside the bracket

Example 2 Expand and simplify $3(x + 5) - 4(2x + 3)$

$3(x + 5) - 4(2x + 3)$ $= 3x + 15 - 8x - 12$ $= 3 - 5x$	**1** Expand each set of brackets separately by multiplying $(x + 5)$ by 3 and $(2x + 3)$ by -4 **2** Simplify by collecting like terms: $3x - 8x = -5x$ and $15 - 12 = 3$

Example 3 Expand and simplify $(x + 3)(x + 2)$

$(x + 3)(x + 2)$ $= x(x + 2) + 3(x + 2)$ $= x^2 + 2x + 3x + 6$ $= x^2 + 5x + 6$	**1** Expand the brackets by multiplying $(x + 2)$ by x and $(x + 2)$ by 3 **2** Simplify by collecting like terms: $2x + 3x = 5x$

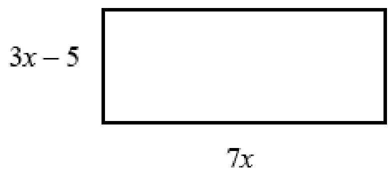
Example 4 Expand and simplify $(x - 5)(2x + 3)$

$(x - 5)(2x + 3)$ $= x(2x + 3) - 5(2x + 3)$ $= 2x^2 + 3x - 10x - 15$ $= 2x^2 - 7x - 15$	**1** Expand the brackets by multiplying $(2x + 3)$ by x and $(2x + 3)$ by -5 **2** Simplify by collecting like terms: $3x - 10x = -7x$

Q1) Expand and simplify where possible.	
a $-2(5pq + 4q^2)$	b $8(5p - 2) - 3(4p + 9)$
c $-2h(6h^2 + 11h - 5)$	d $3b(4b - 3) - b(6b - 9)$
e $(2x + 3)(x - 1)$	f $(3x - 2)(2x + 1)$
g $(5x - 3)(2x - 5)$	h $(3x - 2)(7 + 4x)$
i $(3x + 4y)(5y + 6x)$	j $(x + 5)^2$
k $(2x - 7)^2$	l $(4x - 3y)^2$

Q2) Problem Solving

a) The diagram shows a rectangle.
Write down an expression, in terms of x, for the area of the rectangle.
Show that the area of the rectangle can be written as $21x^2 - 35x$.



b) A cuboid has dimensions (x + 2)cm, (2x – 1)cm and (2x + 3)cm.
Show that the volume of the cuboid is $(4x^3 + 12x^2 + 5x - 6)cm^3$

Answers - Expanding brackets and simplifying expressions

1 **a** $-10pq - 8q^2$

b $40p - 16 - 12p - 27 = 28p - 43$

c $10h - 12h^3 - 22h^2$

d $6b^2$

e $2x^2 + x - 3$

f $6x^2 - x - 2$

g $10x^2 - 31x + 15$

h $12x^2 + 13x - 14$

i $18x^2 + 39xy + 20y^2$

j $x^2 + 10x + 25$

k $4x^2 - 28x + 49$

l $16x^2 - 24xy + 9y^2$

2 **a** $7x(3x - 5) = 21x^2 - 35x$

b $(4x^3 + 12x^2 + 5x - 6)cm^3$

Surds and rationalising the denominator

Key points

- A surd is the square root of a number that is not a square number, for example $\sqrt{2}, \sqrt{3}, \sqrt{5},$ etc.
- Surds can be used to give the exact value for an answer.
- $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$
- $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$
- To rationalise the denominator means to remove the surd from the denominator of a fraction.
- To rationalise $\frac{a}{\sqrt{b}}$ you multiply the numerator and denominator by the surd $\sqrt{b}$
- To rationalise $\frac{a}{b+\sqrt{c}}$ you multiply the numerator and denominator by $b-\sqrt{c}$

Examples

Example 1 Simplify $\sqrt{50}$

$\sqrt{50} = \sqrt{25 \times 2}$	**1** Choose two numbers that are factors of 50. One of the factors must be a square number
$= \sqrt{25} \times \sqrt{2}$	**2** Use the rule $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$
$= 5 \times \sqrt{2}$	**3** Use $\sqrt{25} = 5$
$= 5\sqrt{2}$	

Example 2 Simplify $\sqrt{147} - 2\sqrt{12}$

$\sqrt{147} - 2\sqrt{12}$ $= \sqrt{49 \times 3} - 2\sqrt{4 \times 3}$	**1** Simplify $\sqrt{147}$ and $2\sqrt{12}$. Choose two numbers that are factors of 147 and two numbers that are factors of 12. One of each pair of factors must be a square number
	2 Use the rule $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$
$= \sqrt{49} \times \sqrt{3} - 2\sqrt{4} \times \sqrt{3}$	**3** Use $\sqrt{49} = 7$ and $\sqrt{4} = 2$
$= 7 \times \sqrt{3} - 2 \times 2 \times \sqrt{3}$	
$= 7\sqrt{3} - 4\sqrt{3}$	**4** Collect like terms
$= 3\sqrt{3}$	

Example 3 Simplify $\left(\sqrt{7}+\sqrt{2}\right)\left(\sqrt{7}-\sqrt{2}\right)$

$\left(\sqrt{7}+\sqrt{2}\right)\left(\sqrt{7}-\sqrt{2}\right)$ $= \sqrt{49} - \sqrt{7}\sqrt{2} + \sqrt{2}\sqrt{7} - \sqrt{4}$	**1** Expand the brackets. A common mistake here is to write $\left(\sqrt{7}\right)^2 = 49$
$= 7 - 2$ $= 5$	**2** Collect like terms: $-\sqrt{7}\sqrt{2} + \sqrt{2}\sqrt{7}$ $= -\sqrt{7}\sqrt{2} + \sqrt{7}\sqrt{2} = 0$

Example 4 Rationalise $\frac{1}{\sqrt{3}}$

$\frac{1}{\sqrt{3}} = \frac{1}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$	**1** Multiply the numerator and denominator by $\sqrt{3}$
$= \frac{1 \times \sqrt{3}}{\sqrt{9}}$	**2** Use $\sqrt{9} = 3$
$= \frac{\sqrt{3}}{3}$	

Example 5 Rationalise and simplify $\frac{\sqrt{2}}{\sqrt{12}}$

$\frac{\sqrt{2}}{\sqrt{12}} = \frac{\sqrt{2}}{\sqrt{12}} \times \frac{\sqrt{12}}{\sqrt{12}}$	**1** Multiply the numerator and denominator by $\sqrt{12}$
$= \frac{\sqrt{2} \times \sqrt{4 \times 3}}{12}$	**2** Simplify $\sqrt{12}$ in the numerator. Choose two numbers that are factors of 12. One of the factors must be a square number
	3 Use the rule $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$
$= \frac{2\sqrt{2}\sqrt{3}}{12}$	**4** Use $\sqrt{4} = 2$
$= \frac{\sqrt{2}\sqrt{3}}{6}$	**5** Simplify the fraction: $\frac{2}{12}$ simplifies to $\frac{1}{6}$

Example 6 Rationalise and simplify $\frac{3}{2+\sqrt{5}}$

$\frac{3}{2+\sqrt{5}} = \frac{3}{2+\sqrt{5}} \times \frac{2-\sqrt{5}}{2-\sqrt{5}}$	**1** Multiply the numerator and denominator by $2-\sqrt{5}$
$= \frac{3\left(2-\sqrt{5}\right)}{\left(2+\sqrt{5}\right)\left(2-\sqrt{5}\right)}$	**2** Expand the brackets
$= \frac{6-3\sqrt{5}}{4+2\sqrt{5}-2\sqrt{5}-5}$	**3** Simplify the fraction
$= \frac{6-3\sqrt{5}}{-1}$	**4** Divide the numerator by −1 Remember to change the sign of all terms when dividing by −1
$= 3\sqrt{5}-6$	

Q1) Simplify	
a $\sqrt{45}$	**b** $\sqrt{48}$
c $\sqrt{300}$	**d** $\sqrt{72}$
e $\sqrt{72}+\sqrt{162}$	**f** $\sqrt{50}-\sqrt{8}$
g $2\sqrt{28}+\sqrt{28}$	**h** $2\sqrt{12}-\sqrt{12}+\sqrt{27}$

Q2) Expand and Simplify	
a $(\sqrt{2}+\sqrt{3})(\sqrt{2}-\sqrt{3})$	**b** $(4-\sqrt{5})(\sqrt{45}+2)$

Q3) Rationalise and simplify, if possible.	
a $\frac{1}{\sqrt{5}}$	**b** $\frac{2}{\sqrt{7}}$
c $\frac{2}{\sqrt{2}}$	**d** $\frac{\sqrt{8}}{\sqrt{24}}$
e $\frac{2}{4+\sqrt{3}}$	**f** $\frac{6}{5-\sqrt{2}}$

Q4) Problem Solving
Solve the equation $8+x\sqrt{12}=\frac{8x}{\sqrt{3}}$. Give your answer in the form $a\sqrt{b}$ where a and b are integers.

Answers - Surds and rationalising the denominator

1 a $3\sqrt{5}$

b $4\sqrt{3}$

c $10\sqrt{3}$

d $6\sqrt{2}$

e $15\sqrt{2}$

f $3\sqrt{2}$

g $6\sqrt{7}$

h $5\sqrt{3}$

2 a -1

b $10\sqrt{5}-7$

3 a $\frac{\sqrt{5}}{5}$

b $\frac{2\sqrt{7}}{7}$

c $\sqrt{2}$

d $\frac{\sqrt{3}}{3}$

e $\frac{2(4-\sqrt{3})}{13}$

f $\frac{6(5+\sqrt{2})}{23}$

4 $4\sqrt{3}$

<u>**Rules of indices**</u>

Key points

- $a^m \times a^n = a^{m+n}$
- $\dfrac{a^m}{a^n} = a^{m-n}$
- $(a^m)^n = a^{mn}$
- $a^0 = 1$
- $a^{\frac{1}{n}} = \sqrt[n]{a}$ i.e. the *n*th root of *a*
- $a^{\frac{m}{n}} = \sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m$
- $a^{-m} = \dfrac{1}{a^m}$
- The square root of a number produces two solutions, e.g. $\sqrt{16} = \pm 4$.

Examples

Example 1 Evaluate 10^0

$10^0 = 1$	Any value raised to the power of zero is equal to 1

Example 2 Evaluate $9^{\frac{1}{2}}$

$9^{\frac{1}{2}} = \sqrt{9}$ $= 3$	Use the rule $a^{\frac{1}{n}} = \sqrt[n]{a}$

Example 3 Evaluate $27^{\frac{2}{3}}$

$27^{\frac{2}{3}} = \left(\sqrt[3]{27}\right)^2$ $= 3^2$ $= 9$	**1** Use the rule $a^{\frac{m}{n}} = \left(\sqrt[n]{a}\right)^m$ **2** Use $\sqrt[3]{27} = 3$

Example 4 Evaluate 4^{-2}

$4^{-2} = \dfrac{1}{4^2}$ $= \dfrac{1}{16}$	**1** Use the rule $a^{-m} = \dfrac{1}{a^m}$ **2** Use $4^2 = 16$

Example 5 Simplify $\frac{6x^5}{2x^2}$

$\frac{6x^5}{2x^2} = 3x^3$	$6 \div 2 = 3$ and use the rule $\frac{a^m}{a^n} = a^{m-n}$ to give $\frac{x^5}{x^2} = x^{5-2} = x^3$

Example 6 Simplify $\frac{x^3 \times x^5}{x^4}$

$\frac{x^3 \times x^5}{x^4} = \frac{x^{3+5}}{x^4} = \frac{x^8}{x^4}$ $= x^{8-4} = x^4$	**1** Use the rule $a^m \times a^n = a^{m+n}$ **2** Use the rule $\frac{a^m}{a^n} = a^{m-n}$

Example 7 Write $\frac{1}{3x}$ as a single power of x

$\frac{1}{3x} = \frac{1}{3}x^{-1}$	Use the rule $\frac{1}{a^m} = a^{-m}$, note that the fraction $\frac{1}{3}$ remains unchanged

Example 8 Write $\frac{4}{\sqrt{x}}$ as a single power of x

$\frac{4}{\sqrt{x}} = \frac{4}{x^{\frac{1}{2}}}$ $= 4x^{-\frac{1}{2}}$	**1** Use the rule $a^{\frac{1}{n}} = \sqrt[n]{a}$ **2** Use the rule $\frac{1}{a^m} = a^{-m}$

Q1) Evaluate	
a 14^0	b $125^{\frac{1}{3}}$
c $8^{\frac{5}{3}}$	d 2^{-5}
e $27^{-\frac{2}{3}}$	f $\left(\frac{9}{16}\right)^{-\frac{1}{2}}$
g $16^{\frac{1}{4}} \times 2^{-3}$	h $\left(\frac{27}{64}\right)^{-\frac{2}{3}}$

Q2) Simplify	
a $\dfrac{10x^5}{2x^2 \times x}$	b $\dfrac{c^{\frac{1}{2}}}{c^2 \times c^{\frac{3}{2}}}$
c $\dfrac{y^2}{y^{\frac{1}{2}} \times y}$	d $\dfrac{\left(2x^2\right)^3}{4x^0}$

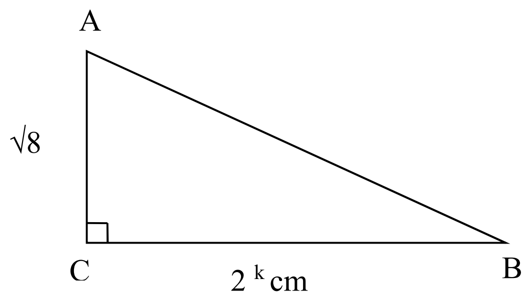
Q3) Write the following as a single power of *x*.	
a $\dfrac{1}{x^7}$	b $\dfrac{1}{\sqrt[3]{x}}$
c $\sqrt[5]{x^2}$	d $\dfrac{1}{\sqrt[3]{x^2}}$

Q4) Write the following in the form ax^n.	
a $\dfrac{2}{x^3}$	b $\dfrac{4}{\sqrt[3]{x}}$

Q5) Problem Solving

a) Triangle ABC has an area of 32 cm^2.

Calculate the value of k.



b) Given that $243\sqrt{3} = 3^a$, find the value of a.

Answers - Rules of indices

1 **a** 1

b 5

c 32

d $\frac{1}{32}$

e $\frac{1}{9}$

f $\frac{4}{3}$

g $\frac{1}{4}$

h $\frac{16}{9}$

2 **a** $5x^2$

b c^{-3}

c $y^{\frac{1}{2}}$

d $2x^6$

3 **a** x^{-7}

e $x^{-\frac{1}{3}}$

d $x^{\frac{2}{5}}$

f $x^{-\frac{2}{3}}$

4 **a** $2x^{-3}$

b $4x^{-\frac{1}{3}}$

5 **a** $k = 4.5$ **b** $a = \frac{11}{2}$

Factorising expressions

Key points

- Factorising an expression is the opposite of expanding the brackets.
- A quadratic expression is in the form $ax^2 + bx + c$, where $a \neq 0$.
- To factorise a quadratic equation find two numbers whose sum is b and whose product is ac.
- An expression in the form $x^2 - y^2$ is called the difference of two squares. It factorises to $(x - y)(x + y)$.

Examples

Example 1 Factorise $15x^2y^3 + 9x^4y$

$15x^2y^3 + 9x^4y = 3x^2y(5y^2 + 3x^2)$	The highest common factor is $3x^2y$. So take $3x^2y$ outside the brackets and then divide each term by $3x^2y$ to find the terms in the brackets

Example 2 Factorise $4x^2 - 25y^2$

$4x^2 - 25y^2 = (2x + 5y)(2x - 5y)$	This is the difference of two squares as the two terms can be written as $(2x)^2$ and $(5y)^2$

Example 3 Factorise $x^2 + 3x - 10$

$b = 3$, $ac = -10$	**1** Work out the two factors of $ac = -10$ which add to give $b = 3$ (5 and −2)
So $x^2 + 3x - 10 = x^2 + 5x - 2x - 10$	**2** Rewrite the b term ($3x$) using these two factors
$= x(x + 5) - 2(x + 5)$	**3** Factorise the first two terms and the last two terms
$= (x + 5)(x - 2)$	**4** $(x + 5)$ is a factor of both terms

Example 4 Factorise $6x^2 - 11x - 10$

$b = -11$, $ac = -60$	**1** Work out the two factors of $ac = -60$ which add to give $b = -11$ (−15 and 4)
So $6x^2 - 11x - 10 = 6x^2 - 15x + 4x - 10$	**2** Rewrite the b term ($-11x$) using these two factors
$= 3x(2x - 5) + 2(2x - 5)$	**3** Factorise the first two terms and the last two terms
$= (2x - 5)(3x + 2)$	**4** $(2x - 5)$ is a factor of both terms

Example 5 Simplify $\dfrac{x^2-4x-21}{2x^2+9x+9}$

$\dfrac{x^2-4x-21}{2x^2+9x+9}$	**1** Factorise the numerator and the denominator
For the numerator: $b = -4$, $ac = -21$	**2** Work out the two factors of $ac = -21$ which add to give $b = -4$ (-7 and 3)
So $x^2 - 4x - 21 = x^2 - 7x + 3x - 21$	**3** Rewrite the b term ($-4x$) using these two factors
$= x(x - 7) + 3(x - 7)$	**4** Factorise the first two terms and the last two terms
$= (x - 7)(x + 3)$	**5** $(x - 7)$ is a factor of both terms
For the denominator: $b = 9$, $ac = 18$	**6** Work out the two factors of $ac = 18$ which add to give $b = 9$ (6 and 3)
So $2x^2 + 9x + 9 = 2x^2 + 6x + 3x + 9$	**7** Rewrite the b term ($9x$) using these two factors
$= 2x(x + 3) + 3(x + 3)$	**8** Factorise the first two terms and the last two terms
$= (x + 3)(2x + 3)$	**9** $(x + 3)$ is a factor of both terms
So $\dfrac{x^2-4x-21}{2x^2+9x+9} = \dfrac{(x-7)(x+3)}{(x+3)(2x+3)}$ $= \dfrac{x-7}{2x+3}$	**10** $(x + 3)$ is a factor of both the numerator and denominator so cancels out as a value divided by itself is 1

Q1) Factorise	
a $25x^2y^2 - 10x^3y^2 + 15x^2y^3$	b $18a^2 - 200b^2c^2$
c $x^2 - 3x - 40$	d $x^2 + 3x - 28$
e $2x^2 + 7x + 3$	f $12x^2 - 38x + 20$

Q2) Simplify the algebraic fractions	
a $\dfrac{2x^2+4x}{x^2-x}$	**b** $\dfrac{x^2-x-12}{x^2-4x}$
c $\dfrac{x^2-5x}{x^2-25}$	**d** $\dfrac{2x^2-7x-15}{3x^2-17x+10}$
e $\dfrac{4-25x^2}{10x^2-11x-6}$	**f** $\dfrac{(x+2)^2+3(x+2)^2}{x^2-4}$

Q3) Problem Solving

a) Express

$\dfrac{1}{x-2} - \dfrac{2}{x+4}$ as a single algebraic fraction.

b) Hence, or otherwise, solve

$$\frac{1}{x-2} - \frac{2}{x+4} = \frac{1}{3}$$

Answers - Factorising expressions

1 **a** $5x^2y^2(5 - 2x + 3y)$

b $2(3a - 10bc)(3a + 10bc)$

c $(x - 8)(x + 5)$

d $(x + 7)(x - 4)$

e $(2x + 1)(x + 3)$

f $2(3x - 2)(2x - 5)$

2 **a** $\frac{2(x+2)}{x-1}$

b $\frac{x+3}{x}$

c $\frac{x}{x+5}$

d $\frac{2x+3}{3x-2}$

e $\frac{2-5x}{2x-3}$

f $\frac{4(x+2)}{x-2}$

3 a $\frac{8-x}{(x-2)(x+4)}$

b $x = -8.68$, 3.68

Solving quadratic equations by factorisation

Key points

- A quadratic equation is an equation in the form $ax^2 + bx + c = 0$ where $a \neq 0$.
- To factorise a quadratic equation find two numbers whose sum is b and whose products is ac.
- When the product of two numbers is 0, then at least one of the numbers must be 0.
- If a quadratic can be solved it will have two solutions (these may be equal).

Examples

Example 1 Solve $5x^2 = 15x$

$5x^2 = 15x$ $5x^2 - 15x = 0$	**1** Rearrange the equation so that all of the terms are on one side of the equation and it is equal to zero. Do not divide both sides by x as this would lose the solution $x = 0$.
$5x(x - 3) = 0$	**2** Factorise the quadratic equation. $5x$ is a common factor.
So $5x = 0$ or $(x - 3) = 0$	**3** When two values multiply to make zero, at least one of the values must be zero.
Therefore $x = 0$ or $x = 3$	**4** Solve these two equations.

Example 2 Solve $x^2 + 7x + 12 = 0$

$x^2 + 7x + 12 = 0$ $b = 7$, $ac = 12$	**1** Factorise the quadratic equation. Work out the two factors of $ac = 12$ which add to give you $b = 7$. (4 and 3)
$x^2 + 4x + 3x + 12 = 0$	**2** Rewrite the b term ($7x$) using these two factors.
$x(x + 4) + 3(x + 4) = 0$	**3** Factorise the first two terms and the last two terms.
$(x + 4)(x + 3) = 0$	**4** $(x + 4)$ is a factor of both terms.
So $(x + 4) = 0$ or $(x + 3) = 0$	**5** When two values multiply to make zero, at least one of the values must be zero.
Therefore $x = -4$ or $x = -3$	**6** Solve these two equations.

Example 3 Solve $9x^2 - 16 = 0$

$9x^2 - 16 = 0$ $(3x + 4)(3x - 4) = 0$	**1** Factorise the quadratic equation. This is the difference of two squares as the two terms are $(3x)^2$ and $(4)^2$.
So $(3x + 4) = 0$ or $(3x - 4) = 0$	**2** When two values multiply to make zero, at least one of the values must be zero.
$x = -\frac{4}{3}$ or $x = \frac{4}{3}$	**3** Solve these two equations.

Example 4 Solve $2x^2 - 5x - 12 = 0$

$b = -5$, $ac = -24$	**1** Factorise the quadratic equation. Work out the two factors of $ac = -24$ which add to give you $b = -5$. (-8 and 3)
So $2x^2 - 8x + 3x - 12 = 0$	**2** Rewrite the b term ($-5x$) using these two factors.
$2x(x - 4) + 3(x - 4) = 0$	**3** Factorise the first two terms and the last two terms.
$(x - 4)(2x + 3) = 0$	**4** $(x - 4)$ is a factor of both terms.
So $(x - 4) = 0$ or $(2x + 3) = 0$	**5** When two values multiply to make zero, at least one of the values must be zero.
$x = 4$ or $x = -\frac{3}{2}$	**6** Solve these two equations.

Solving quadratic equations by completing the square

Key points

- Completing the square lets you write a quadratic equation in the form $p(x + q)^2 + r = 0$.

Examples

Example 5 Solve $x^2 + 6x + 4 = 0$. Give your solutions in surd form.

$x^2 + 6x + 4 = 0$	
$(x + 3)^2 - 9 + 4 = 0$	**1** Write $x^2 + bx + c = 0$ in the form $\left(x + \frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2 + c = 0$
$(x + 3)^2 - 5 = 0$	**2** Simplify.
$(x + 3)^2 = 5$	**3** Rearrange the equation to work out x. First, add 5 to both sides.
$x + 3 = \pm\sqrt{5}$	**4** Square root both sides. Remember that the square root of a value gives two answers.
$x = \pm\sqrt{5} - 3$	**5** Subtract 3 from both sides to solve the equation.
So $x = -\sqrt{5} - 3$ or $x = \sqrt{5} - 3$	**6** Write down both solutions.

Example 6 Solve $2x^2 - 7x + 4 = 0$. Give your solutions in surd form.

$2x^2 - 7x + 4 = 0$	**1** Before completing the square write $ax^2 + bx + c$ in the form $a\left(x^2 + \frac{b}{a}x\right) + c$
$2\left(x^2 - \frac{7}{2}x\right) + 4 = 0$	
$2\left[\left(x - \frac{7}{4}\right)^2 - \left(\frac{7}{4}\right)^2\right] + 4 = 0$	**2** Now complete the square by writing $x^2 - \frac{7}{2}x$ in the form $\left(x + \frac{b}{2a}\right)^2 - \left(\frac{b}{2a}\right)^2$
$2\left(x - \frac{7}{4}\right)^2 - \frac{49}{8} + 4 = 0$	**3** Expand the square brackets.
$2\left(x - \frac{7}{4}\right)^2 - \frac{17}{8} = 0$	**4** Simplify.
	(continued on next page)
$2\left(x - \frac{7}{4}\right)^2 = \frac{17}{8}$	**5** Rearrange the equation to work out x. First, add $\frac{17}{8}$ to both sides.
$\left(x - \frac{7}{4}\right)^2 = \frac{17}{16}$	**6** Divide both sides by 2.
$x - \frac{7}{4} = \pm\frac{\sqrt{17}}{4}$	**7** Square root both sides. Remember that the square root of a value gives two answers.
$x = \pm\frac{\sqrt{17}}{4} + \frac{7}{4}$	**8** Add $\frac{7}{4}$ to both sides.
So $x = \frac{7}{4} - \frac{\sqrt{17}}{4}$ or $x = \frac{7}{4} + \frac{\sqrt{17}}{4}$	**9** Write down both the solutions.

Solving quadratic equations by using the formula

Key points

- Any quadratic equation of the form $ax^2 + bx + c = 0$ can be solved using the formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
- If $b^2 - 4ac$ is negative then the quadratic equation does not have any real solutions.
- It is useful to write down the formula before substituting the values for a, b and c.

Examples

Example 7 Solve $x^2 + 6x + 4 = 0$. Give your solutions in surd form.

$a = 1, b = 6, c = 4$ $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	**1** Identify a, b and c and write down the formula. Remember that $-b \pm \sqrt{b^2 - 4ac}$ is all over $2a$, not just part of it.
$x = \frac{-6 \pm \sqrt{6^2 - 4(1)(4)}}{2(1)}$	**2** Substitute $a = 1$, $b = 6$, $c = 4$ into the formula.
$x = \frac{-6 \pm \sqrt{20}}{2}$	**3** Simplify. The denominator is 2, but this is only because $a = 1$. The denominator will not always be 2.
$x = \frac{-6 \pm 2\sqrt{5}}{2}$	**4** Simplify $\sqrt{20}$. $\sqrt{20} = \sqrt{4 \times 5} = \sqrt{4} \times \sqrt{5} = 2\sqrt{5}$
$x = -3 \pm \sqrt{5}$	**5** Simplify by dividing numerator and denominator by 2.
So $x = -3 - \sqrt{5}$ or $x = \sqrt{5} - 3$	**6** Write down both the solutions.

Example 8 Solve $3x^2 - 7x - 2 = 0$. Give your solutions in surd form.

$a = 3, b = -7, c = -2$ $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	**1** Identify a, b and c, making sure you get the signs right and write down the formula. Remember that $-b \pm \sqrt{b^2 - 4ac}$ is all over $2a$, not just part of it.
$x = \frac{-(-7) \pm \sqrt{(-7)^2 - 4(3)(-2)}}{2(3)}$	**2** Substitute $a = 3$, $b = -7$, $c = -2$ into the formula.
$x = \frac{7 \pm \sqrt{73}}{6}$	**3** Simplify. The denominator is 6 when $a = 3$. A common mistake is to always write a denominator of 2.
So $x = \frac{7 - \sqrt{73}}{6}$ or $x = \frac{7 + \sqrt{73}}{6}$	**4** Write down both the solutions.

Q1) Solve by factorising	
a $x^2 + 7x + 10 = 0$	b $x^2 + 3x - 10 = 0$
c $2x^2 - 7x - 4 = 0$	d $3x^2 - 13x - 10 = 0$
Q2) Solve by completing the square	
a $x^2 - 10x + 4 = 0$	b $x^2 - 2x - 6 = 0$
c $2x^2 + 8x - 5 = 0$	d $5x^2 + 3x - 4 = 0$
Q3) Solve by using the quadratic formula, giving your solutions in surd form	
a $3x^2 + 6x + 2 = 0$	b $2x^2 - 4x - 7 = 0$
Q4) Problem Solving	

$x^2 - 14x + 1 = (x + p)^2 + q$, where p and q are constants.

a Find the values of p and q.

b Using your answer to part **a**, or otherwise, show that the solutions to the equation $x^2 - 14x + 1 = 0$ can be written in the form $r \pm s\sqrt{3}$, where r and s are constants to be found.

Q5) Problem Solving

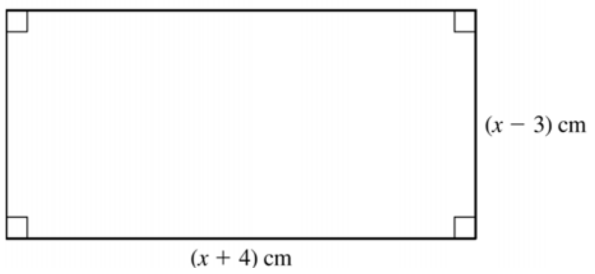


Diagram **NOT** accurately drawn.

The length of a rectangle is ($x + 4$) cm. The width is ($x - 3$) cm. The area of the rectangle is 78 cm^2.

(a) Use this information to write down an equation in terms of x.

(b) (i) Show that your equation in part **(a)** can be written as

$$x^2 + x - 90 = 0$$

(ii) Find the values of x which are the solutions of the equation

$$x^2 + x - 90 = 0$$

x = ... or x = ...

(iii) Write down the length and the width of the rectangle.

length = ... cm width = ... cm

Answers – Solving Quadratics

1 **a** $x = -5$ or $x = -2$

b $x = -5$ or $x = 2$

c $x = -\frac{1}{2}$ or $x = 4$

d $x = -\frac{2}{3}$ or $x = 5$

2 **a** $x = 5 + \sqrt{21}$ or $x = 5 - \sqrt{21}$

b $x = 1 + \sqrt{7}$ or $x = 1 - \sqrt{7}$

c $x = -2 + \sqrt{6.5}$ or $x = -2 - \sqrt{6.5}$

d $x = \frac{-3+\sqrt{89}}{10}$ or $x = \frac{-3-\sqrt{89}}{10}$

3 **a** $x = -1 + \frac{\sqrt{3}}{3}$ or $x = -1 - \frac{\sqrt{3}}{3}$

b $x = 1 + \frac{3\sqrt{2}}{2}$ or $x = 1 - \frac{3\sqrt{2}}{2}$

4

a $p = -7$, $q = -48$

b $(x - 7)^2 = 48$

$x = 7 \pm \sqrt{48} = 7 \pm 4\sqrt{3}$

$r = 7$, $s = 4$

5 a $(x + 4)(x - 3) = 78$

b (i) As given

(ii) 9 or −10

(iii) 13 cm, 6 cm